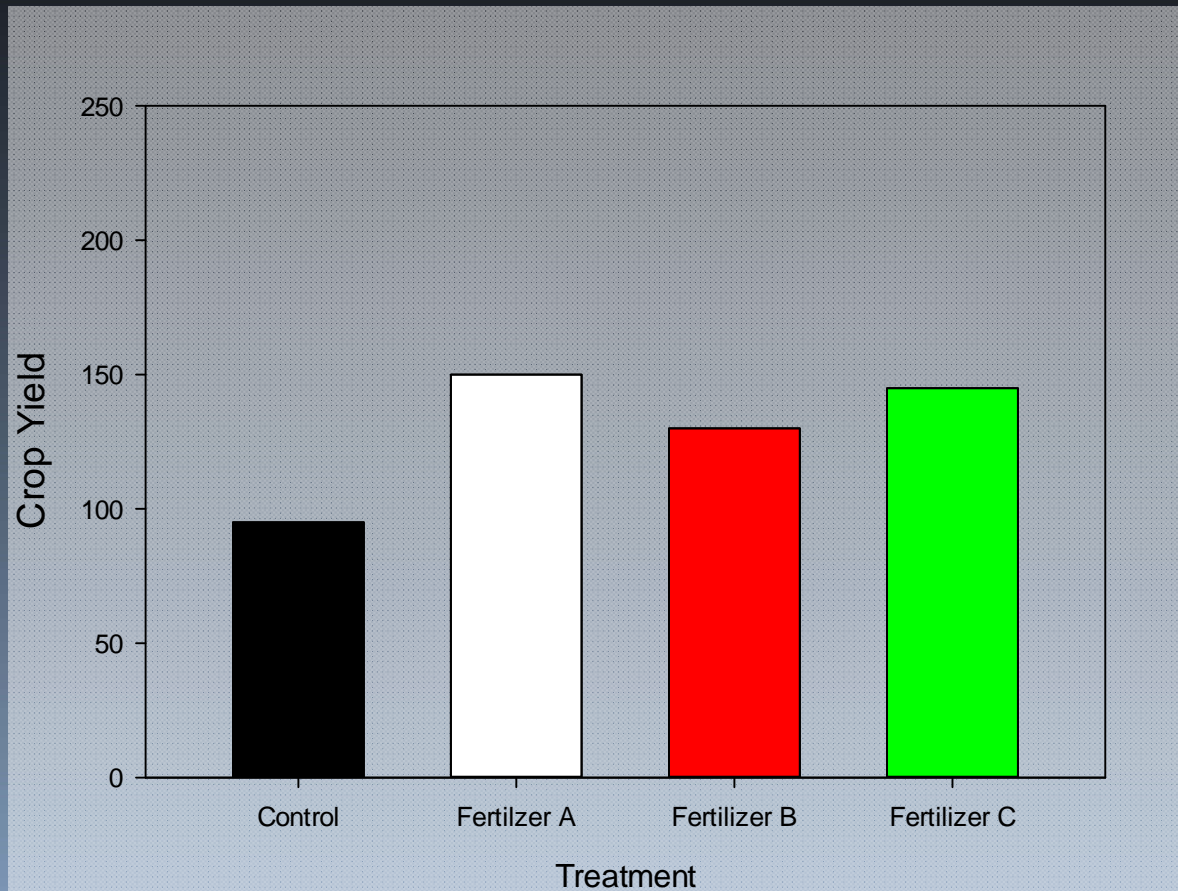
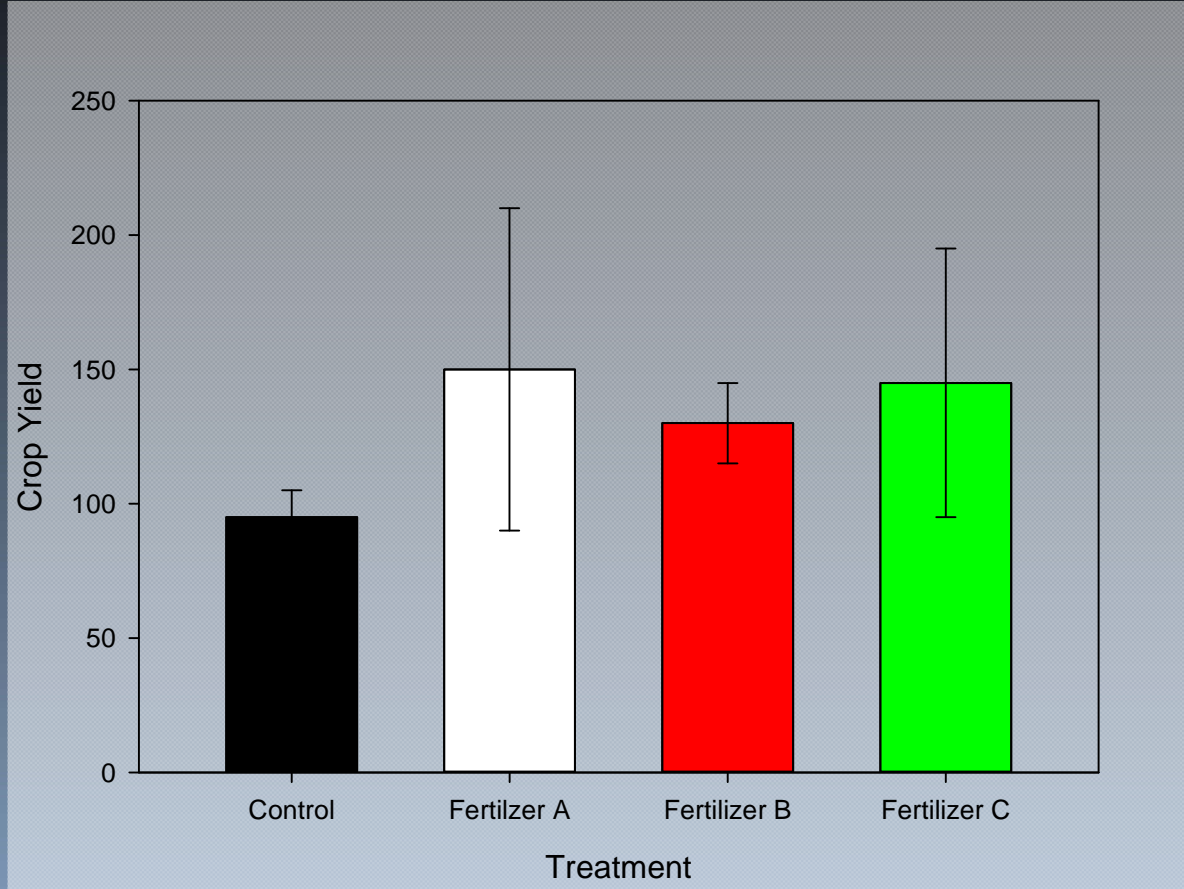
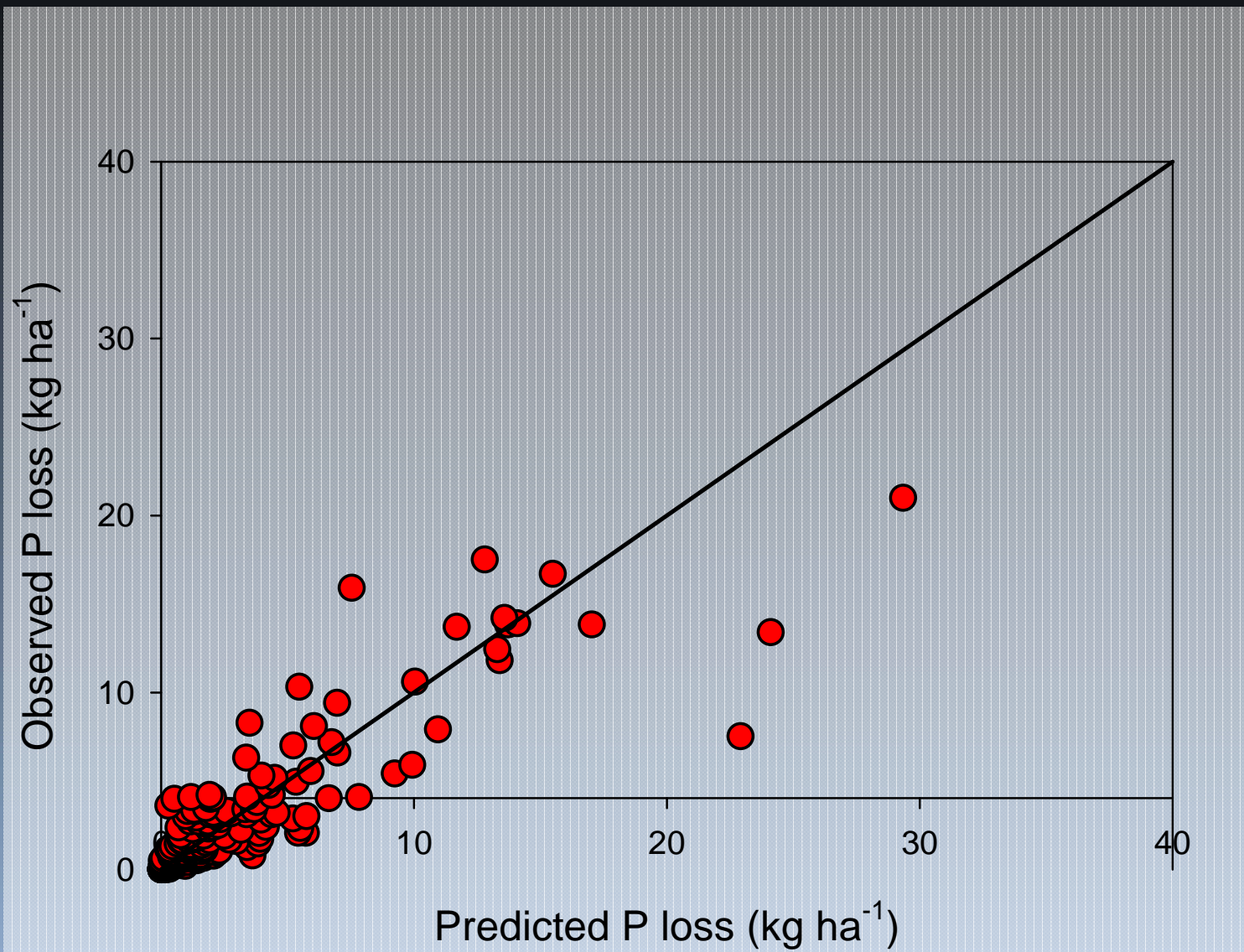


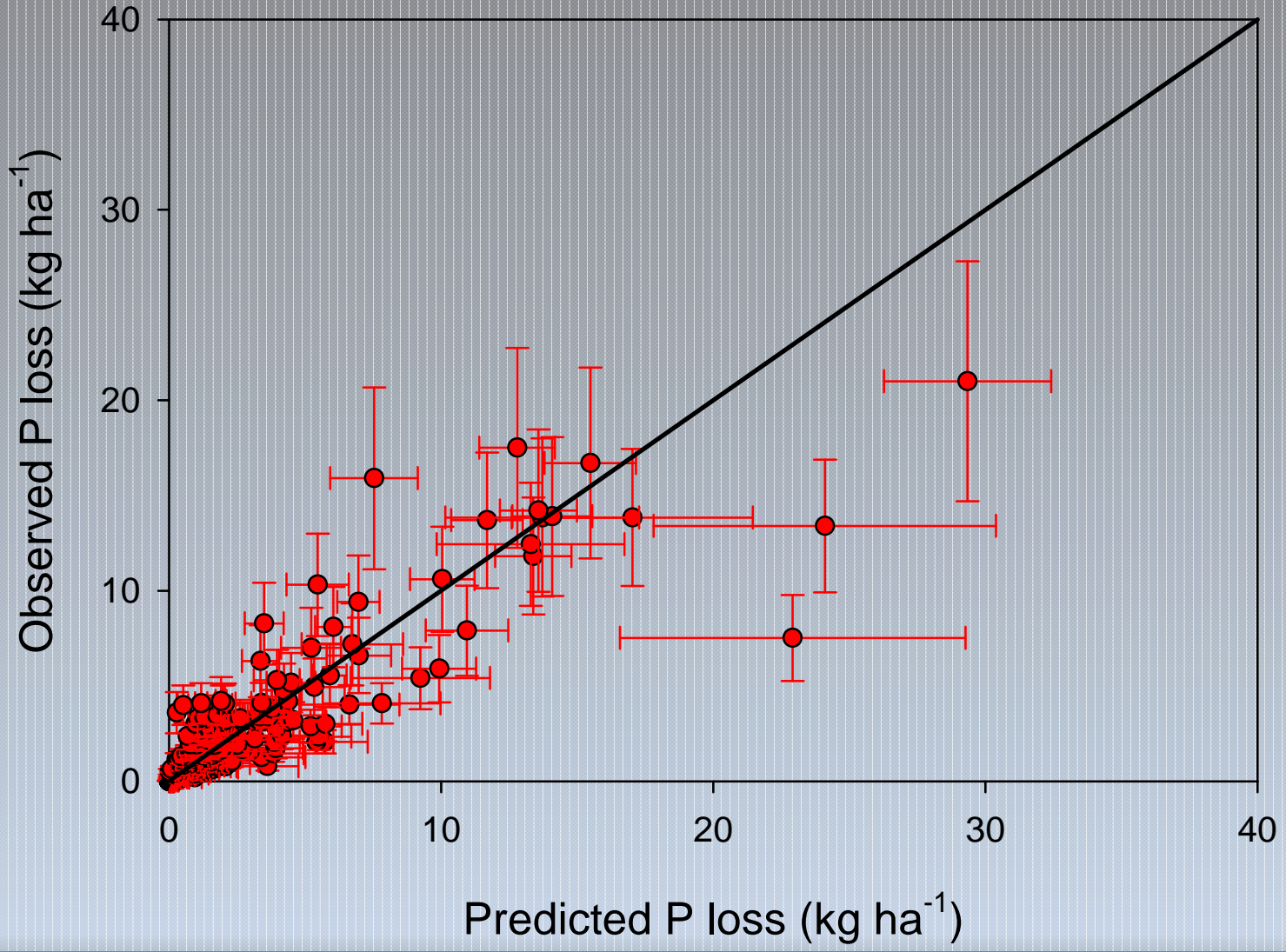
The Need for Model Uncertainty Analysis

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Uncertainty about uncertainties

- Hard to calculate
- Many sources...which ones to focus on?
- Lack of training/expertise/access to uncertainty analysis
- How to communicate to lay audience?
- Fear that uncertainties may undermine credibility of the model

Sources of Model Uncertainty

- Model structure error
 - All models are approximations
 - “All models are wrong, some are useful”
- Model input error (variables such as rainfall, soil test P)
 - Measurement errors
 - Unrepresentative values
- Model parameter error (Generally obtained through calibration)
 - Incorrect optimization targets
 - Inaccurate, incomplete, or unrepresentative calibration data

Objectives

1. Estimate the uncertainty associated with five regression equations used in APLE
2. Compare relative magnitudes of parameter and model input uncertainties

APLE model equations

$$P_{\text{tot}} = P_{\text{sed}} + DP_{\text{soil}} + DP_{\text{man}} + DP_{\text{fert}}$$

P_{tot} is the total annual P loss from surface runoff (kg ha^{-1}),

P_{sed} is annual sediment P loss from eroded soil (kg ha^{-1}),

DP_{soil} is annual DRP loss in runoff from soil (kg ha^{-1}),

DP_{man} is annual DRP loss in runoff from applied manure (kg ha^{-1}),

DP_{fert} is annual DRP loss in runoff from applied fertilizer (kg ha^{-1}).

APLE model equations

$$P_{\text{sed}} = ER \cdot PER \cdot TP \cdot 10^{-6}$$

$$PER = C_1 \cdot ER^{C_2}$$

$$TP = LP \cdot (1 - PAI) / PAI + 4(LP \cdot (1 - PAI) / PAI) + OP$$

$$PAI = -C_3 \cdot \ln(\%Clay) + C_4 \cdot LP - C_5 \cdot SOC + C_6$$

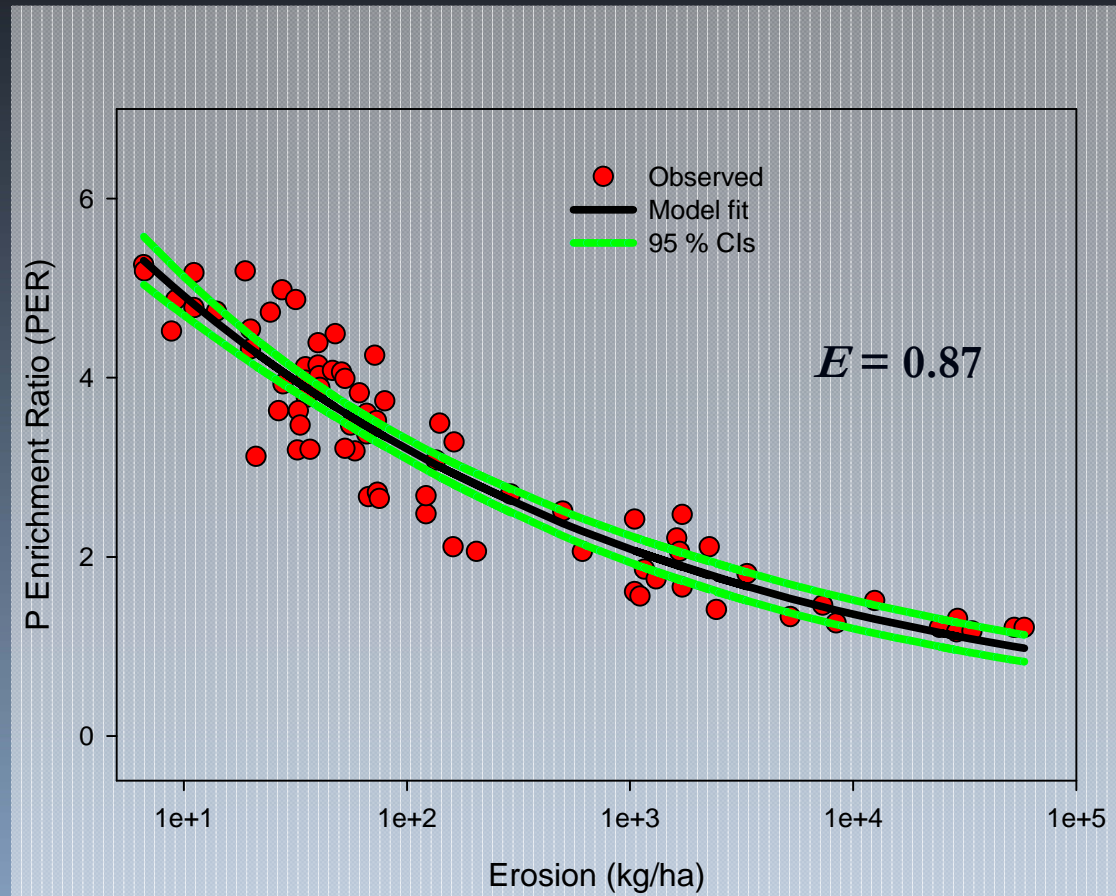
ER is the annual erosion rate (kg ha⁻¹),
PER is the P enrichment ratio
TP is total soil P (mg kg⁻¹),

LP is labile P,
OP is organic P,
%Clay is clay content of soil,
SOC is soil organic carbon

Methods: Objective 1

- Fit data using least squares regression
- Calculated 95 % confidence intervals (CIs) for the five equations
 - Represent uncertainty associated with mean response of the model
- Calculated 95 % prediction intervals
 - Accounts for parameter uncertainty AND unexplained variability
 - Used for making predictions for a single observation

P enrichment ratio (PER) fits and uncertainties

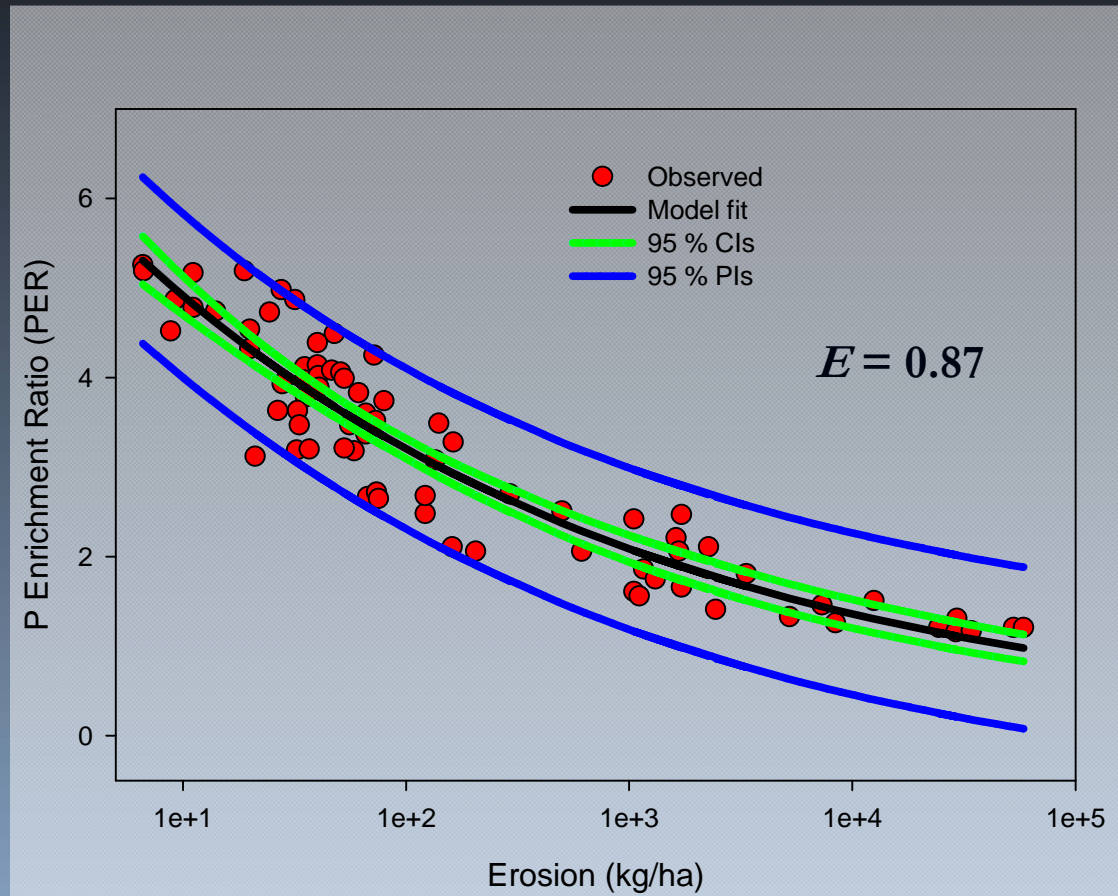


| CV | |
|-------|------|
| C_1 | 4.3% |
| C_2 | 5.7% |

CI $\pm 3.1 - 15 \%$

Data from Sharpley. 2007. *In* Radcliffe and Cabrera, eds., Modeling Phosphorus in the Environment.

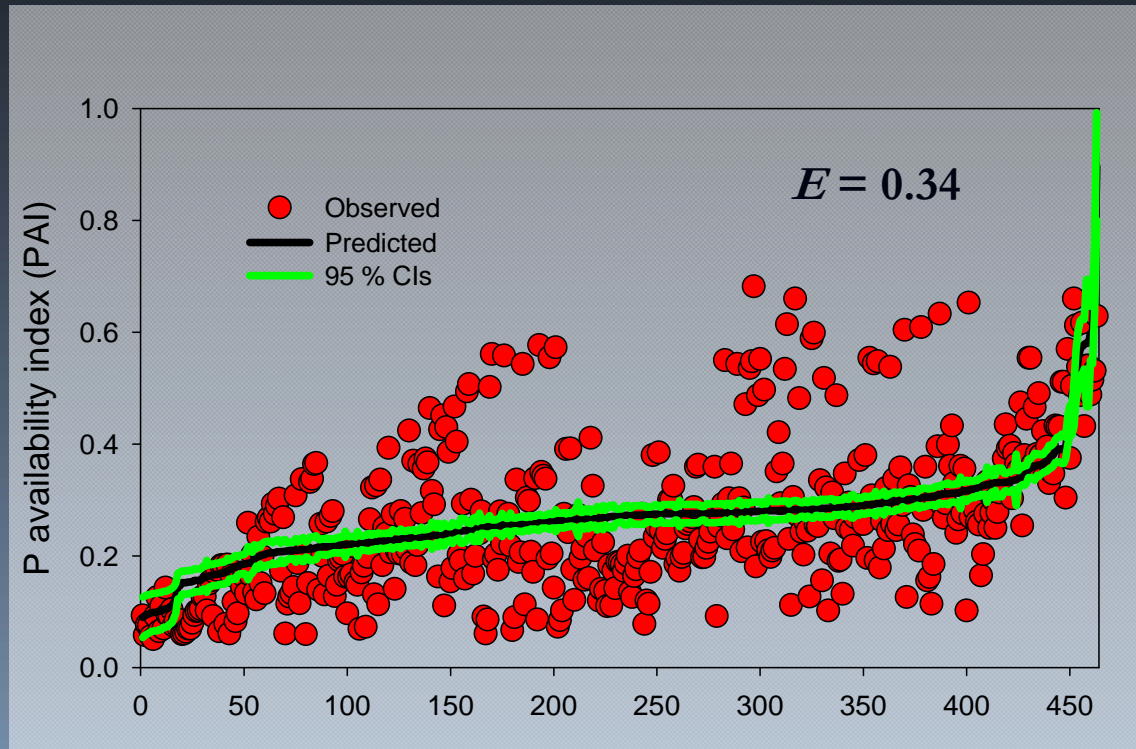
P enrichment ratio (PER) fits and uncertainties



| | |
|----|-------------------|
| CI | $\pm 3.1 - 15 \%$ |
| PI | $\pm 17 - 92 \%$ |

Data from Sharpley. 2007. *In* Radcliffe and Cabrera, eds., Modeling Phosphorus in the Environment.

P availability index (PAI) fits and uncertainties

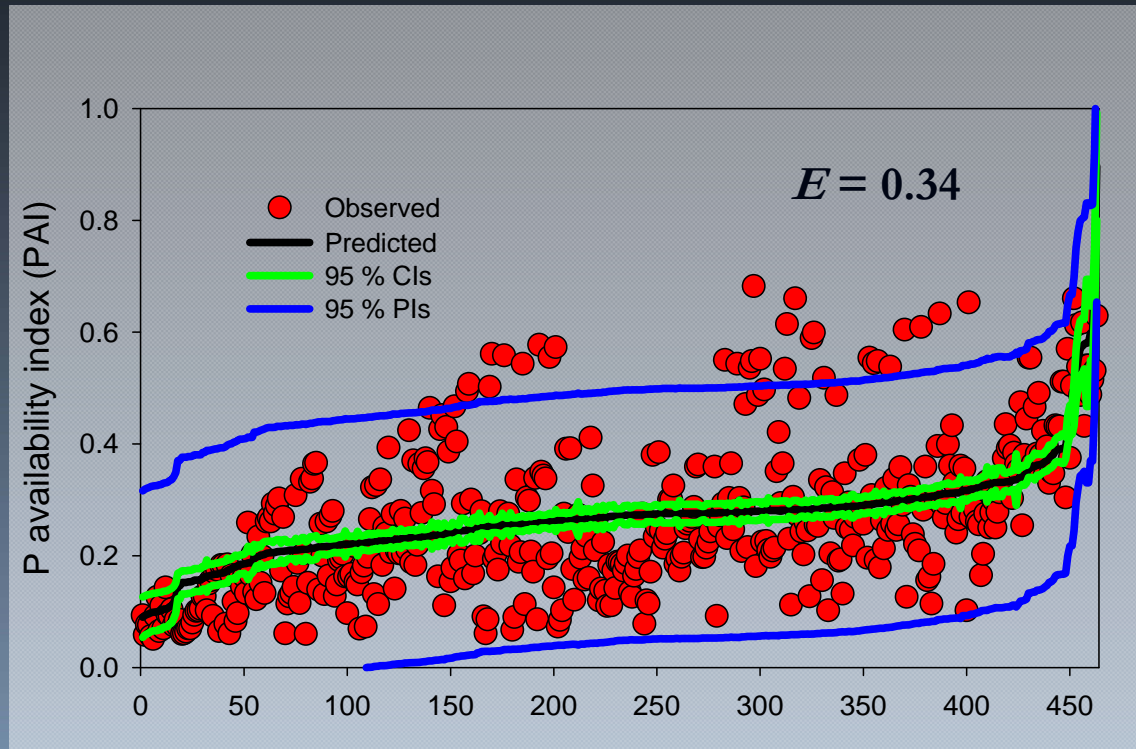


| | CV |
|-------|------|
| C_3 | 16% |
| C_4 | 15% |
| C_5 | 7.2% |
| C_6 | 5.9% |

CI $\pm 3.9 - 40\%$

Data from Vadas and White. 2010. TASABE 53: 1469-1476.

P availability index (PAI) fits and uncertainties



CI $\pm 3.9 - 40\%$
PI $\pm 25 - 250\%$

Data from Vadas and White. 2010. TASABE 53: 1469-1476.

APLE model equations

$$P_{\text{sed}} = \text{ER} \cdot \text{PER} \cdot \text{TP} \cdot 10^{-6}$$

$$\text{PER} = C_1 \cdot \text{ER}^{C_2}$$

$$\text{TP} = \text{LP} \cdot (1 - \text{PAI}) / \text{PAI} + 4(\text{LP} \cdot (1 - \text{PAI}) / \text{PAI}) + \text{OP}$$

$$\text{PAI} = -C_3 \cdot \ln(\% \text{Clay}) + C_4 \cdot \text{LP} - C_5 \cdot \text{SOC} + C_6$$

ER is the annual erosion rate (kg ha^{-1}),
PER is the P enrichment ratio
TP is total soil P (mg kg^{-1}),

LP is labile P,
OP is organic P,
%Clay is clay content of soil,
SOC is soil organic carbon

APLE input variables

| P Loss Pathway | | | |
|------------------------------|----------------------------------|---------------|-----------------------------|
| DP_{man} | DP_{fert} | DP_{soil} | P_{sed} |
| Runoff/Precip | Runoff/Precip | Runoff | Soil Loss |
| Total manure applied | Total P applied | Soil Labile P | Soil Labile P |
| Percent manure solids | Percent fertilizer incorporation | | Soil clay content |
| Manure TP content | | | Soil organic matter content |
| Water extractable P | | | |
| Percent manure incorporation | | | |
| Mineralization rate | | | |

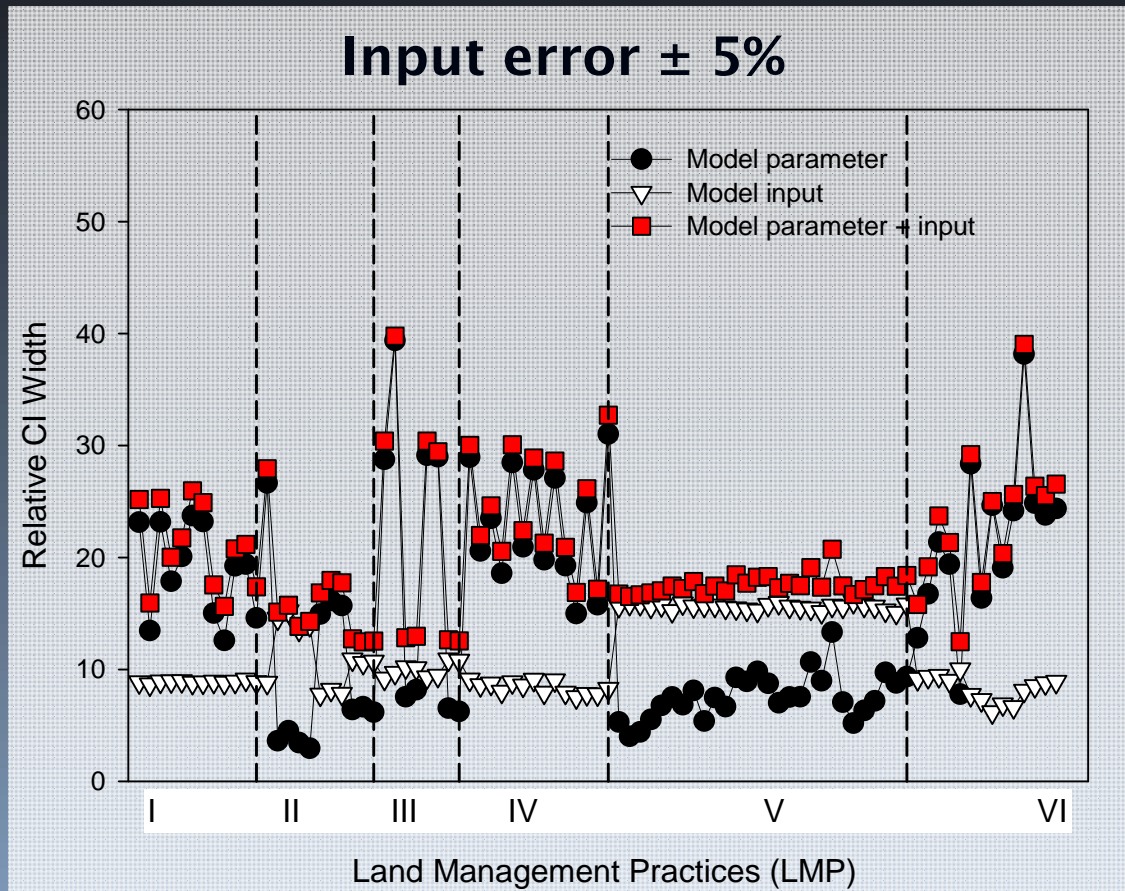
Assumed Errors in Model Input Variables

| Model Variable | Small Uncertainty | Large Uncertainty |
|-----------------------|----------------------|-----------------------|
| Runoff (weir) | $\pm 5\%$ | $\pm 10\%$ |
| Runoff (direct) | $\pm 1\%$ | $\pm 3\%$ |
| Erosion | $\pm 2.5\%$ | $\pm 5\%$ |
| Manure mineralization | $\pm 2.5\%$ | $\pm 5\%$ |
| P incorporation rates | $\pm 5\%$ (constant) | $\pm 10\%$ (constant) |
| All other variables | $\pm 5\%$ | $\pm 15\%$ |

Based on Harmel et al. 2006

Results: Objective 2

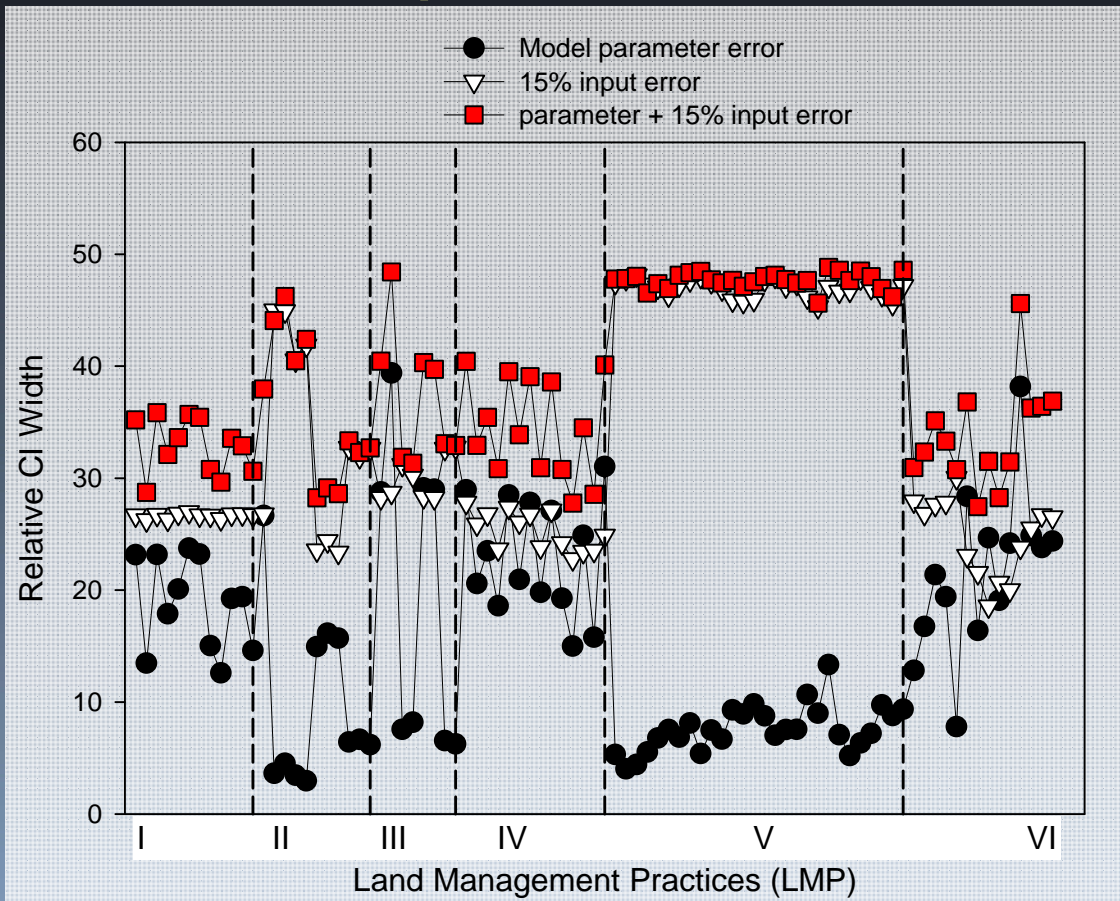
Contribution of model parameter and input error



- I) no P applied
- II) inorganic fertilizer,
- III) manure to fields
w/o erosion
- IV) manure to fields
with erosion,
- V) fertilizer and
manure to fields
w/o erosion
- VI) fertilizer and
manure to fields
with erosion.

Contribution of model parameter and input error

Input error $\pm 15\%$

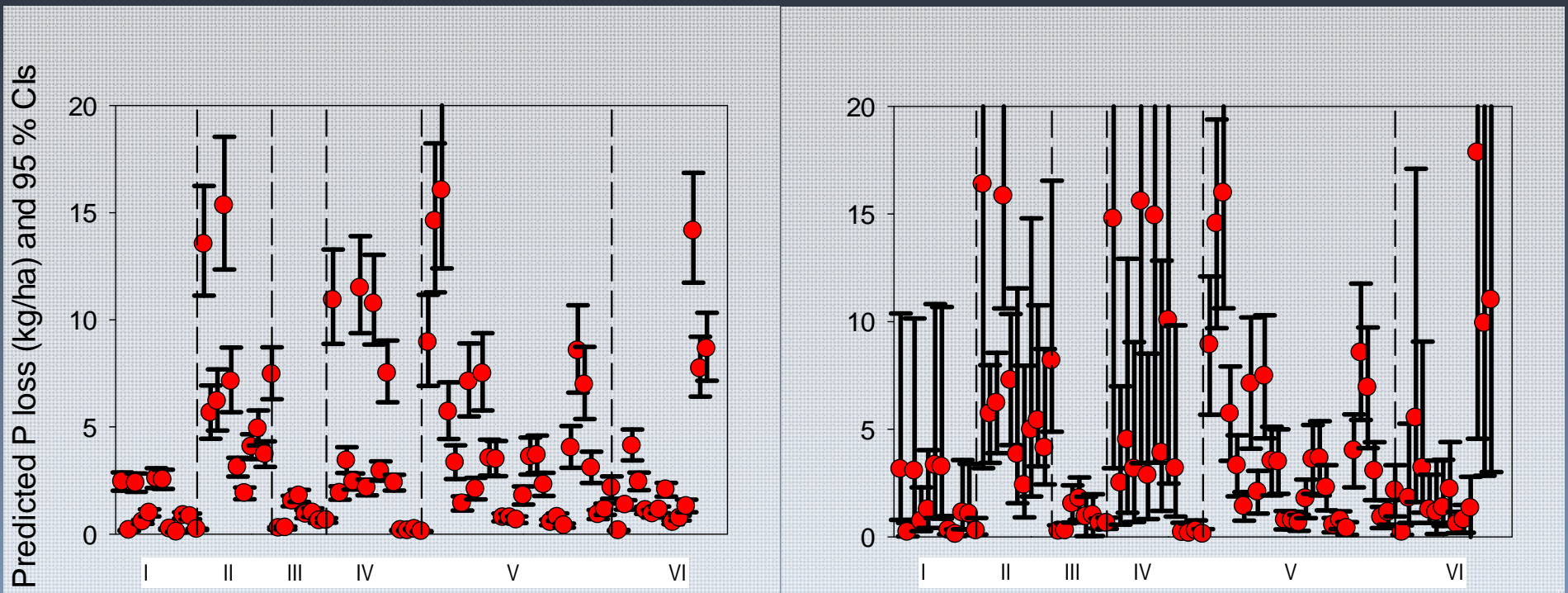


- I) no P applied
- II) inorganic fertilizer,
- III) manure to fields w/o erosion
- IV) manure to fields with erosion,
- V) fertilizer and manure to fields w/o erosion
- VI) fertilizer and manure to fields with erosion.

Model prediction uncertainties

Model Input + Parameter
Uncertainties (95 % CIs)

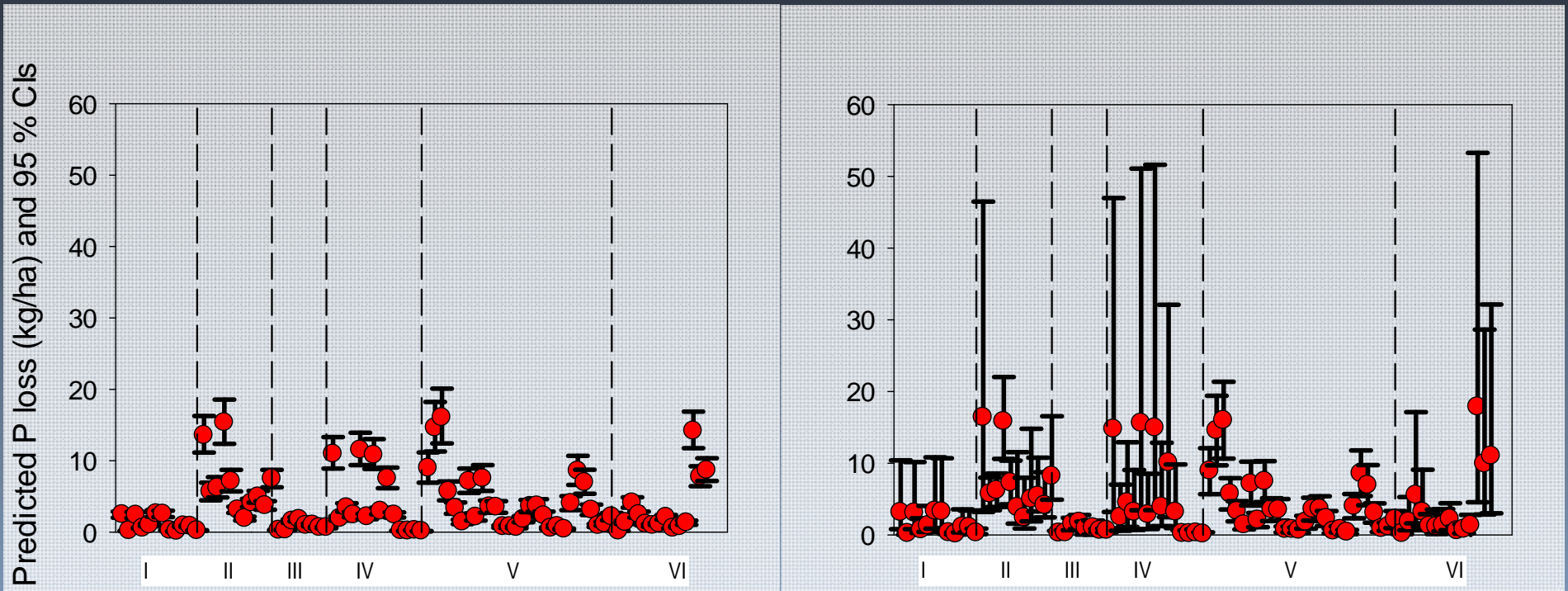
95 % PIs



Model prediction uncertainties

Model Input + Parameter
Uncertainties (95 % CIs)

95 % PIs



Conclusions

- **Uncertainties in model predictions are a fact of life**
 - Ignoring them may do more harm than good
- **Uncertainties in model predictions can help us better evaluate our models**
- **As modelers it is our responsibility to faithfully present the limitations with our model predictions to our audience**
- **“Doubt is not a pleasant condition, but certainty is absurd.” VOLTAIRE**

Questions?



This research was conducted as part of USDA-ARS National Program 214: Agricultural and Industrial By-Products